DE MORGAN TO SYLVESTER.

THE most extraordinary personage for half a century in the mathematical world was James Joseph, self-styled Sylvester.

The wittiest, most permanently charming of English logicians is De Morgan, founder of the now-towering Logic of Relatives.

No word of Sylvester's about De Morgan has ever been recorded; nor any word of De Morgan's about Sylvester except the brief sentence in a letter to Dean Peacock, November 13, 1838: "You are, of course, interested in all that concerns Dr. Young. A publisher in London has bought or will buy the plates of the *Lectures*. He proposes to republish them (catalogue excepted) in parts, transferring the copper to lithograph. My colleague Sylvester is to put notes, which *with reading* he will do very well."

Once the name occurs in the opening sentence of a letter to Herschel, May 30, 1862:

"My dear Sir John,—I should not wonder if Sylvester and you were at one without any intercommunication of your particles."

That is all.

Yet Sylvester was for a short time a pupil of De Morgan (1828).

There was only eight years difference in their ages, though De Morgan was dead and gone before Sylvester even began his creation of our Western Continent for mathematics.

The pupil, after a decade, became (1837–1838) the colleague, by appointment to the chair of Natural Philosophy in University College, London.

This official connexion was sufficiently brief. Sylvester retired from University College in the session 1840–1841, and immediately

afterwards accepted the Professorship of Mathematics in the University of Virginia.

Of his six months in the wilds of Charlottesville and of his exit thence, least said, soonest mended. The Virginians so utterly failed to understand Sylvester, his character, his aspirations, his superhuman powers, that the Rev. Dr. R. L. Dabney, of Virginia, has seriously assured me that Sylvester was actually deficient in intellect, a sort of semi-idiotic calculating boy!

On the 15th of September, 1855, Sylvester took up the appointment of Professor of Mathematics at the Royal Military Academy, Woolwich, and to him there were written the precious letters from De Morgan which form the final cause of this brief paper.

He occupied K Quarters, Woolwich Common, being the last of a long list of residential professors.

De Morgan's first letter makes it probable that at first, temporarily, he occupied A Quarters on the Common. His permanent residence was a commodious house with a good garden. There, seated under a walnut tree, he made some of the greatest of those marvellous discoveries and creations with which he has enriched all time. Would that the memorial medal, to be awarded triennially by the Royal Society, could have pictured his face lighted with such "solving-the-universe." Instead, the mood it shows is of those periods "wasted in fighting the world."

Not even the existence of the following letters has ever before been known to the world.

7 Camden Street, January 8, 1856.

MY DEAR SYLVESTER.

Can it be *a* the Common? Your mark is A very distinct. Have you given up living at a specific number, and taken to an algebraical symbol?

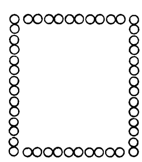
You will certainly be directed to at

$$\begin{vmatrix} a, & b, & c \\ a', & b', & c' \\ a'', & b'', & c'' \end{vmatrix}$$

if you go on in this way.

I am very glad you like your new quarters. Your simile of Egyptian bondage reminds me that Lincoln's Inn Fields is said to be exactly the size of the base of the great pyramid.

Now some have said that your ancestors built the pyramids in the days when they were making bricks without straw. See the old book which a living speculator (the man who invented the



machine for the postage stamps) says was originally written in *Greek*, the language of *Canaan*, and afterwards translated into Hebrew.

I hope, when you went away, you killed all their first-born, especially such as were insured in the office. Nevermind, *renovare dolorem* is one of the pleasures of life—and you are now *professionally* occupied when you handle a determinant.

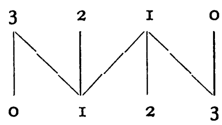
You will smile when you look at the answer to the puzzle.

Who has laid down what they used to call the *affections* of *oblique* spherical triangles, meaning the relations of acute and obtuse? I cannot find anybody. Now the law of these things is as follows;—

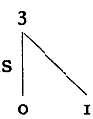
In every oblique angled spherical triangle, *either* each side is of the same *name* (acute or obtuse) with its opposite angle, or an *odd* number of *acute* sides coexists with an *odd* number of *obtuse* angles.

And conversely, every case contained under the preceding exists, except that *three* acute sides cannot exist with three obtuse angles.

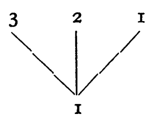
All this is figured in the diagram I sent you where the upper



numerals denote the number of *sides* which are *acute*, and the lower ones the number of ANGLES which are OBTUSE.

Thus 

denotes that three acute sides coexist with none or one obtuse angle.



denotes that one angle (only) obtuse consists with either 3, 2, or 1 acute sides.

But 

denotes that with two sides acute (and one obtuse) there must be one, and only one, obtuse angle.

For so small a matter this is useful enough.

What I want to find out is whether anybody has ever taken the trouble to collect the cases, and to state the result.

I am pretty sure that neither Delambre, Puissant, Legendre, Cagnoli, nor T. S. Davies, has done it. And these are the mathematicians I know who have paid most attention to spherical trigonometry in modern times. Yours very truly, A. DE MORGAN.

T. O.

Another actuary has followed you out of that field, but not quite so creditably. Neison is, I understand, hiding from his creditors, and obliged to give up all his posts.

PROBLEM. If a professor at Woolwich runs away from his wife, and his successor takes his house, required to ascertain how far the same successor is bound to take the wife too?

[The Differential Calculus is excluded.]

7 Camden Street, Feb. 10, 1856.

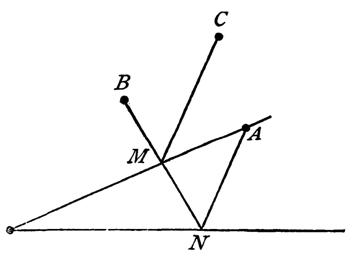
Dear Sylvester:

I send you my exposition of Newton's method, arising out of the Newton-Taylor-Sterling-Stewart-Cramer-Lagrange-Lacroix-Minding-Serret-De Morgan chain of events.

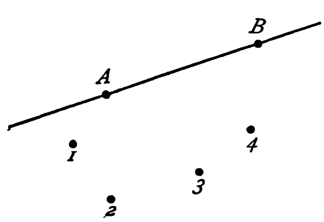
I had been trying to get a really Euclidean demonstration of the proposition that any five points in one plane may be the projection of five points in a circle. All the demonstrations draw a great deal too much upon things visible in algebra but not in geometry.

At every outlet I found a bothering bit of geometry which *just* cut me off. Here was one which I gave to Rutherford, who looked at it, at first with great confidence, as a thing which would be soon finished. But he found himself stopped.

Can you manage it simply?



Given two lines, a point A in one of them, and *any* two other points B, C. Required to draw BMN cutting the two lines in M and N so that CM, AN are parallel. The problem is certainly Euclidean, for it leads to an equation of the second degree with one unknown quantity. I will not allow you to assume that four points



1, 2, 3, 4, not in a straight line, cannot have more than *two equi-anharmonic* pencils with their vertices in one straight line. This

is another way out of the original question, and therefore is, I doubt not, connected with the problem I give.

Whence came the word *anharmonic?* It is Chasles's, I believe. Is *an* the ἀνά or the ἀν privative?

I am used to the word *enharmonic* in music, which I suspect is just as badly derived. Yours very truly,

A. De Morgan.

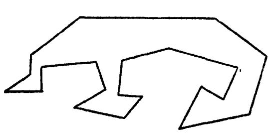
7 Camden Street, April 11, 1856.

Dear Sylvester:

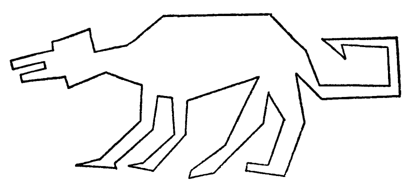
At last I return you your quarter squares, with whole thanks. They go by this post. Newman tells me to-day he hears the journal is abandoned—is it true?

The following connexion between the partition of numbers and the formation of autotomic polygons struck me a few days ago. I find it tames that interminable list of hexagons which one has to draw in Pascal's theorem. (N. B. It is a great pity he had two A's in his name. If he had been Pescal, the hexagon PESCAL would have been a *fait établi.*)

1. Learn to denote a simple polygon—not autotomic—as

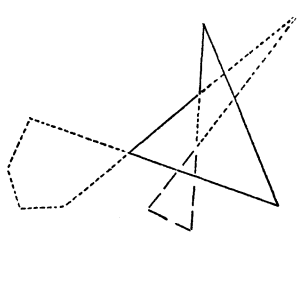


or



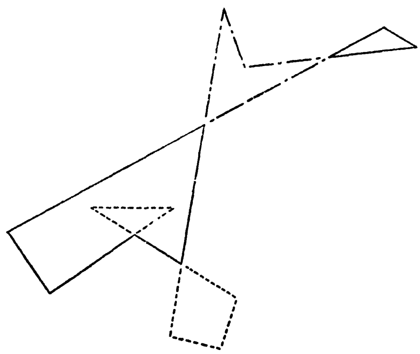
by the number of pairs of right angles in the sum of its internal angles, $m-2$ for an m—gon. Let $m-2$ be the type.

2. Every autotomic polygon is formed by simple polygons joined at opposite angles, as



And the type of the whole is the sum of the types of its parts.

The type of this figure



is $3\begin{cases}2\{1\\1\\2\end{cases}$ a simple pentagon with two tetragons and a triangle at three angles, and a triangle joined to one of the tetragons.

Or the type may be $2\begin{cases}3\begin{cases}1\\2\end{cases}\\1\end{cases}$, $1\{3\begin{cases}2\{1\\2\end{cases}$, $2\{3\begin{cases}2\{1\\1\end{cases}$, or $1\{2\{3\begin{cases}1\\2\end{cases}$

The figure is a $\{(3+2+1+2+1)+2\}$-gon.

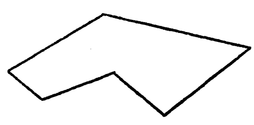
All the partitions of $m-2$ are 2^{m-3} in number. If one of these be $a_1+a_2+a_3+$, the types arising are $a_1\{a_2\{a_3\{\ldots.$

$$a_1\begin{cases}a_2\{a_4\{a_5\ldots. \text{ etc., etc.}\\a_3\end{cases}$$

The essentially different types of a hexagon are

$$4,\ 3\{1,\ 2\{1\{1,\ 2\begin{cases}1\\1\end{cases},\ 1\{1\{1\{1,\ 1\begin{cases}1\\1\\1\end{cases},\ 2\{2.$$

4



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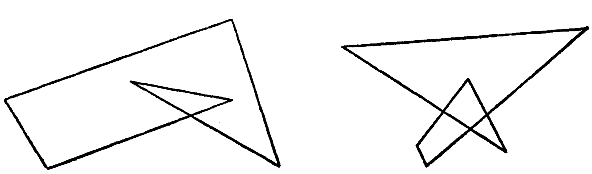
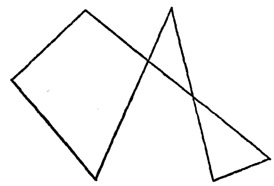
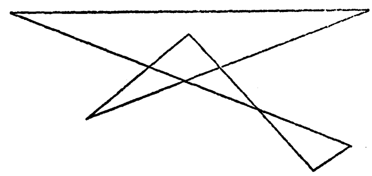


2(1(1



$2\begin{cases}1\\1\end{cases}$



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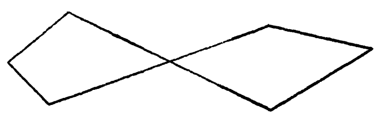
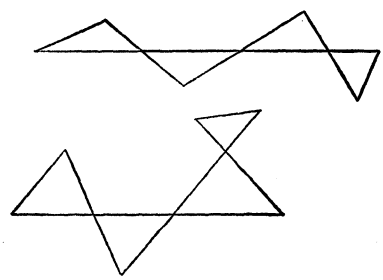
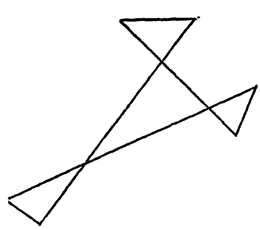


1(1(1(1



$1\begin{cases}1\\1\\1\end{cases}$



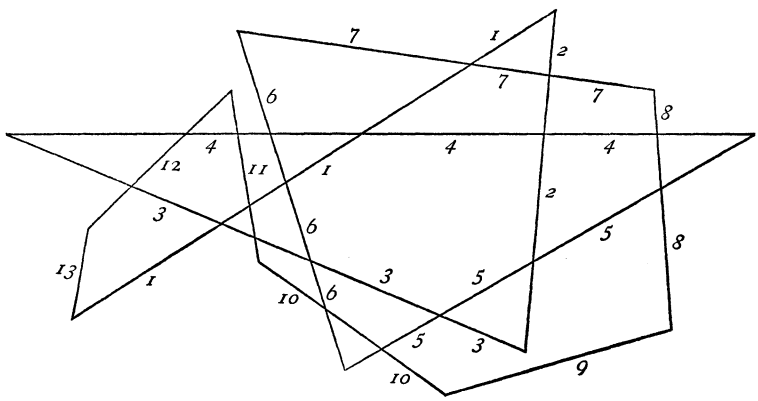
There are many sub-species arising out of choice of angles to join at, angles salient or re-entrant, and sides longer or shorter, so as to affect the mutual sections of sides.

But this is the first rating.

A paper on these polygons might be written without a dia-

gram: leaving every one to draw his own diagrams by the following symbolic description.

Let 1, 2, 3, etc., be the sides of a polygon; in order and between m and $m+1$ write all the sides which $m+1$ cuts, of lower name. Thus 1, 2_1, 3, 4 is a tetragon in which 3 cuts 1.



This polygon might be recovered by the description

$$1\ 2_1\ 3_{12}\ 4_{23}\ 5_{31}\ 6_{12}\ 7_{45}\ 8\ 9_{56}\ 10_{314}\ 11_{43}\ 12\ 13.$$

What symbols are possible and what impossible, I do not know. You are just the man to find out. Yours very truly,

A. DE MORGAN.

91 Adelaide Road, May 14, 1865.

DEAR SYLVESTER:

I am glad to hear that you think the theorem n-true. How Newton got it, I never can venture to imagine. He was a very *plodding* man: quite capable of constructing an equation to the 20th degree by given roots, and trying the theorem upon it.

Newton used to try numerical values upon series, by way of verification, and said he should be ashamed to say how far he had gone in this way, and how much time he had spent.

Airy told me—when I was an undergraduate—that no demonstration ever persuaded him that $x - \frac{x^3}{2\cdot 3} + \frac{x^5}{2\cdot 3\cdot 4\cdot 5} - \dots$ was

periodic in value, until he tried it upon numbers of some value, as $x = 10$. Yours very truly, A. DE MORGAN.

* * *

Myself loving De Morgan and feasting on his books, especially his deliciously witty *Budget of Paradoxes*, I naturally asked Sylvester for *his* judgment. He answered me in an epigram :

> "De Morgan did not write mathematics;
> He wrote *about* mathematics!"

GEORGE BRUCE HALSTED.

AUSTIN, TEXAS.